

FJ started a 30 day treatment program which involved daily injections of a medication. The first day's injection was 12 mg, and each subsequent day's injection was 5% less than the previous day's injection. Find the total amount of the injections. **SCORE: _____ / 5 PTS**

Your final answer may **NOT** use ... **NOR** \sum . It may use +, -, \times , \div and powers. (It does **NOT** need to be simplified into a single number.)

a_n = AMOUNT OF INJECTION ON n^{TH} DAY

$$a_1 = 12$$

$$a_2 = 12(1 - 0.05) = 12(0.95)$$

$$a_3 = 12(0.95)^2$$

\vdots

$$a_n = 12(0.95)^{n-1}$$

$$\text{TOTAL} = \frac{\overset{\textcircled{1}}{12} \overset{\textcircled{2\frac{1}{2}}}{(1 - 0.95^{30})}}{\underset{\textcircled{1\frac{1}{2}}}{1 - 0.95}}$$

Use Pascal's Triangle to expand and simplify $(n^3 - 2r^4)^5$.

SCORE: ____ / 5 PTS

$$(n^3)^5 + 5(n^3)^4(-2r^4) + 10(n^3)^3(-2r^4)^2 + 10(n^3)^2(-2r^4)^3 + 5(n^3)(-2r^4)^4 + (-2r^4)^5$$

$$= \underbrace{n^{15}}_{\textcircled{\frac{1}{2}}} - \underbrace{10n^{12}r^4}_{\textcircled{1}} + \underbrace{40n^9r^8}_{\textcircled{1}} - \underbrace{80n^6r^{12}}_{\textcircled{1}} + \underbrace{80n^3r^{16}}_{\textcircled{1}} - \underbrace{32r^{20}}_{\textcircled{\frac{1}{2}}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{240} (2 - 5n + 3n^2)$. **SCORE: _____ / 4 PTS**

Your final answer may **NOT** use ... **NOR** \sum . It may use $+$, $-$, \times , \div . (It does **NOT** need to be simplified into a single number.)

$$\sum_{n=1}^{240} 2 - 5 \sum_{n=1}^{240} n + 3 \sum_{n=1}^{240} n^2$$

$$= \underbrace{240 \cdot 2}_{\textcircled{\frac{1}{2}}} - 5 \cdot \underbrace{\frac{240(241)}{2}}_{\textcircled{1\frac{1}{2}}} + 3 \cdot \underbrace{\frac{240(241)(481)}{6}}_{\textcircled{2}}$$

Use mathematical induction to prove that 3 is a factor of $n^3 + 6n^2 + 8n$ for all positive integers n .

SCORE: ____ / 10 PTS

BASIS STEP: 3 IS A FACTOR OF $1^3 + 6(1)^2 + 8(1) = 15$ ①

INDUCTIVE STEP: ASSUME 3 IS A FACTOR OF $k^3 + 6k^2 + 8k$ ①

MUST HAVE
WORD "INTEGER" ①
①

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

[PROVE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 8(k+1)$]

$(k+1)^3 + 6(k+1)^2 + 8(k+1)$ ①

$$= k^3 + 3k^2 + 3k + 1 \\ + 6k^2 + 12k + 6 \\ + 8k + 8$$

$$= (k^3 + 6k^2 + 8k) + (3k^2 + 15k + 15)$$

$= (k^3 + 6k^2 + 8k) + 3(k^2 + 5k + 5)$ ②

SINCE 3 IS A FACTOR OF BOTH $(k^3 + 6k^2 + 8k)$ AND $3(k^2 + 5k + 5)$ ①

THEREFORE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 8(k+1)$ ①

SO, 3 IS A FACTOR OF $n^3 + 6n^2 + 8n$ ⑤/2

FOR ALL INTEGERS $n \geq 1$ ①/2

Find the rational number representation of the repeating decimal 0.427 using the method discussed in lecture.

SCORE: ____ / 6 PTS

NOTE: Only the 27 is repeated.

$$\underline{0.4 + 0.027 + 0.00027 + 0.0000027 + \dots} \quad (1)$$

$$= \frac{4}{10} + \frac{0.027}{1 - \frac{1}{100}}$$

$$= \frac{4}{10} + \boxed{\frac{\frac{27}{1000}}{\frac{99}{100}}} \quad (2\frac{1}{2})$$

$$= \frac{4}{10} + \frac{27}{1000} \cdot \frac{100}{99}$$

$$= \boxed{\frac{4}{10}} + \boxed{\frac{3}{110}} \quad (1)$$

$$= \boxed{\frac{47}{110}} \quad (1)$$